$8+7=3$
$12+1=1$
$10+8=6 \quad 5+21=4$


The result is less than the addends?!?
can other operations also be done?

$$
\begin{array}{lll}
\mathbf{3}-6=9 & 5-3=2 & \mathbf{2}-4=10 \\
6 \times 8=12 & 2 \times 4=8 & 7 \times 5=11
\end{array}
$$



I dlidnet nalke fun of you... You reflect on what I have told you


# In N $8+7=15$, but I wrote 3 ; and so $10+8$ do 18 , but I put 6... 

In which environment did I make these calculations?

$$
\begin{array}{ll}
\operatorname{In} N \ldots \\
8+7=15 & 3 \\
10+8=18 & 6 \\
12+1=13 & 1 \\
5+11=16 & 4
\end{array}
$$



## this is the last help: The time is flowing...

Modular arithmetic

## $100^{12} 1^{1} 2$ 9 $8765^{4}$ <br> CLOCK

## Modular arithmetic



Now we make the table

| + | 0 | ${ }^{1}$ | ${ }^{2}$ | ${ }^{3}$ | 4 | 5 | 6 | 7 | 8 | 9 | ${ }^{10}$ | ${ }^{11}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0 |  |  |  |  |  |  |  |  |  |  |  |  |
| 1 |  |  |  |  |  |  |  |  |  |  |  |  |
| 2 |  |  |  |  |  |  |  |  |  |  |  |  |
| 3 |  |  |  |  |  |  |  |  |  |  |  |  |
| 4 |  |  |  |  |  |  |  |  |  |  |  |  |
| 5 |  |  |  |  |  |  |  |  |  |  |  |  |
| 6 |  |  |  |  |  |  |  |  |  |  |  |  |
| 7 |  |  |  |  |  |  |  |  |  |  |  |  |
| 8 |  |  |  |  |  |  |  |  |  |  |  |  |
| 9 |  |  |  |  |  |  |  |  |  |  |  |  |
| 10 |  |  |  |  |  |  |  |  |  |  |  |  |
| 11 |  |  |  |  |  |  |  |  |  |  |  |  |

## The table of addition

| $A$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 5 | 11 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0 | $Q$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 0 |
| 2 | 2 | 3 | 6 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 5 | 1 |
| 3 | 3 | 4 | 5 | 5 | 7 | 8 | 9 | 10 | 11 | 6 | 1 | 2 |
| 4 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 0 | 1 | 2 | 3 |
| 5 | 6 | 7 | 8 | 9 | 10 | 11 | 0 | 1 | 2 | 3 | 4 |  |
| 6 | 7 | 8 | 9 | 10 | 11 | 6 | 1 | 2 | 3 | 4 | 5 |  |
| 7 | 7 | 8 | 9 | 10 | 11 | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
| 8 | 9 | 10 | 11 | 6 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |  |
| 9 | 10 | 11 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |  |
| 10 | 11 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |  |
| 11 | 2 | 1 | 2 | 3 | 2 | 5 | 6 | 7 | 8 | 9 | 10 |  |
| 1 |  |  |  |  |  |  |  |  |  |  |  |  |

Is the addition close?
Is there a neutral element?
Is there the commutative property?
are the numbers on a straight line? Is there a order?

The table of subtraction

$$
A=\{0 ; 1 ; 2 ; 3 ; 4 ; 5 ; 6 ; 7 ; 8 ; 9 ; 10 ; 11\}
$$

| - | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 2 | 11 | 10 | 3 | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 1 |
| 1 | 11 | 9 | 21 | 10 | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 2 |
| 2 | 10 | 2 | 2 | 11 | 10 | 9 | 8 | 7 | 6 | 5 | 4 | 3 |
| 3 | 9 | 2 | 1 | 2 | 11 | 10 | 9 | 8 | 7 | 6 | 5 | 4 |
| 4 | 8 | 3 | 2 | 1 | 2 | 11 | 10 | 9 | 8 | 7 | 6 | 5 |
| 5 | 7 | 4 | 3 | 2 | 1 | 2 | 11 | 10 | 9 | 8 | 7 | 6 |
| 6 | 6 | 5 | 4 | 3 | 2 | 1 | 2 | 11 | 10 | 9 | 8 | 7 |
| 7 | 5 | 6 | 5 | 4 | 3 | 2 | 1 | 2 | 11 | 10 | 9 | 8 |
| 8 | 4 | 7 | 6 | 5 | 4 | 3 | 2 | 1 | 9 | 11 | 10 | 9 |
| 9 | 3 | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 1 | 2 | 11 | 10 |
| 10 | 2 | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 1 | 0 | 11 |
| 11 | 1 | 10 | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 1 | 0 |

Is the subtraction close?
Is there a neutiral element?
Is there the commutative property?

## Now we do some examples...

The pointer of hours marks 12; what will time mark then 29 hours?
We have to do a division:
29:12 $=2$ with the rest of 5 .
After 29 hours the pointer will do 2 complete turns and it will be on 5 , it's marking 5 o'clock.
In modular awithmetic the operation becomes: $0 \sim 29$

- 5

At 3 : what will time mark then 25 hours?
$3+25=2828: 12=2$ with the rest of 4

The pointer will mark 4 o'clock. $^{\prime}$

## Thanks for yous

 aitention